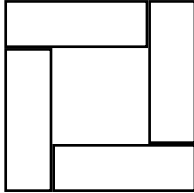





1. One can of frozen juice concentrate, when mixed with $4 \frac{1}{3}$ cans of water, makes 2 quarts (64 oz) of juice. Assuming no volume is gained or lost by mixing, how many oz does a can hold?
- A. 8 B. 10 C. 12 D. 15 E. 18
2. Define the operation Δ by $a \Delta b = ab + b$. Find $(3 \Delta 2) \Delta (2 \Delta 3)$.
- A. 72 B. 73 C. 80 D. 81 E. 90
3. A square is covered by a design made up of four identical rectangles surrounding a central square, as shown at the right. If the area of the central square is $\frac{4}{9}$ of the area of the entire design, find the ratio of the length of a rectangle to the side of the central square.
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- A. $\frac{5}{4}$ B. $\frac{4}{3}$ C. $\frac{7}{5}$ D. $\frac{3}{2}$ E. $\frac{8}{5}$
4. A radio station advertises, "Traffic every 10 minutes, 24 hours a day; 1000 reports each week." What is the difference between the advertised number of reports and the exact number?
- A. 8 B. 12 C. 16 D. 20 E. 24
5. Trina has two dozen coins, all dimes and nickels, worth between \$1.72 and \$2.11. What is the least number of dimes she could have?
- A. 10 B. 11 C. 15 D. 18 E. 19
6. Square $PQRS$ has sides of length 10. Points T , U , V , and W are chosen on sides PQ , QR , RS , and SP respectively so that $PT = QU = RV = SW = 2$. Find the area of quadrilateral $TUVW$.
- A. 48 B. 52 C. 56 D. 64 E. 68
7. A bicycle travels at s feet/min. When its speed is expressed in inches/sec, the numerical value decreases by 16. Find s . (1 foot = 12 inches)
- A. 12 B. 16 C. 18 D. 20 E. 24
8. The average of A and $2B$ is 7, and the average of A and $2C$ is 8. What is the average of A , B , and C ?
- A. 3 B. 4 C. 5 D. 6 E. 9
9. Replace each letter of AMATYC with a digit 0 through 9 to form a six-digit number (identical letters are replaced by identical digits, different letters are replaced by different digits). If the resulting number is the largest such number which is a perfect square, find the sum of its digits (that is, $A + M + A + T + Y + C$).
- A. 32 B. 33 C. 34 D. 35 E. 36
10. A door is 4 ft wide and 7 ft high. If the door is standing open at a 90° angle with the door frame, what is the greatest distance in feet from the outer top corner of the door to a point on the door frame?
- A. 8 B. 9 C. 9.5 D. 10 E. 11

11. A class is exactly 40% female. When 3 male students are replaced by female students, the class becomes exactly 44% female. How many more males than females are in the original class?
A. 10 B. 12 C. 15 D. 18 E. 20
12. A piece has 2 saxophone parts, 3 trumpet parts, and 3 trombone parts. If a band has 2 saxophonists, 3 trumpeters, and 3 trombonists, in how many ways can different parts be assigned to each player?
A. 18 B. 72 C. 324 D. 512 E. 2916
13. Add any integer N to the square of $2N$ to produce an integer M . For how many values of N is M prime?
A. 0 B. 1 C. 2 D. A finite number > 2 E. An infinite number
14. Sixteen students in a dance contest have numbers 1 to 16. When they are paired up, they discover that each couple's numbers add to a perfect square. What is the largest difference between the two numbers for any couple?
A. 5 B. 7 C. 10 D. 12 E. 14
15. In how many distinct ways can a 4×4 square be covered exactly by four  tiles? Assume that rotations and reflections are different coverings.
A. 5 B. 6 C. 8 D. 9 E. 10
16. What is the smallest positive integer that cannot be the degree measure of an exterior angle of a regular polygon?
A. 1 B. 2 C. 3 D. 5 E. 7
17. When certain proper fractions in simplest terms are added, the result is in simplest terms: $\frac{2}{15} + \frac{1}{21} = \frac{19}{105}$; in other cases, the result is not in simplest terms: $\frac{2}{15} + \frac{5}{21} = \frac{39}{105} = \frac{13}{35}$. Assume that $\frac{m}{15}$ and $\frac{n}{21}$ are positive proper fractions in simplest terms. For how many such fractions is $\frac{m}{15} + \frac{n}{21}$ not in simplest terms?
A. 35 B. 48 C. 70 D. 72 E. 140
18. Let r , s , and t be nonnegative integers. How many such triples (r, s, t) satisfy the system $\begin{cases} rs + t = 14 \\ r + st = 13 \end{cases}$?
A. 2 B. 3 C. 4 D. 5 E. 6
19. The average of any 17 consecutive perfect square integers is always k greater than a perfect square. If $k = 2^r m$, where m is odd, find r .
A. 0 B. 1 C. 2 D. 3 E. 4
20. In $\triangle SML$, $SM = 7$ and $ML = 9$. If $\angle M$ is exactly twice as large as $\angle S$, find SL .
A. 10 B. 11 C. 12 D. 13 E. 14

	FALL 2007 ANSWERS
1	C
2	D
3	A
4	A
5	B
6	E
7	D
8	C
9	E
10	B
11	C
12	B
13	C
14	B
15	E
16	E
17	B
18	A
19	D
20	C

1. If $g(x - 1) = x^2 + 1$, find $g(2)$.
A. 1 B. 2 C. 5 D. 9 E. 10
2. Airport runways are labeled by two numbers giving the nonnegative clockwise angles less than 360° of the runway's direction measured from north to the nearest 10° , divided by 10. Thus a runway with heading 223° is labeled 22. What is the other number on this runway?
A. 4 B. 14 C. 16 D. 32 E. 40
3. The equation $a^3 + b^3 + c^3 = 2008$ has a solution in which a , b , and c are distinct even positive integers. Find $a + b + c$.
A. 20 B. 22 C. 24 D. 26 E. 28
4. For how many different integers b is the polynomial $x^2 + bx + 16$ factorable over the integers?
A. 2 B. 3 C. 4 D. 5 E. 6
5. Let $f(x) = x^2 - 2x + 4$. Which of the following is a factor of $f(x) - f(2y)$?
A. $x + 2y$ B. $x + 2y + 2$ C. $x - 2y + 2$ D. $x + 2y - 2$ E. none of these
6. In square $MATH$, M and A lie on a circle of radius 20, and the circle is tangent to side \overline{TH} at the midpoint of \overline{TH} . Find the lengths of the sides of the square.
A. 24 B. 26 C. 28 D. 30 E. 32
7. A fair coin is labeled A on one side and M on the other; a fair die has two sides labeled T, two labeled Y, and two labeled C. The coin and die are each tossed three times. Find the probability that the six letters can be arranged to spell AMATYC.
A. $\frac{1}{60}$ B. $\frac{1}{48}$ C. $\frac{1}{36}$ D. $\frac{1}{24}$ E. $\frac{1}{12}$
8. What is the value of $(\log_{624} 625)(\log_{623} 624)(\log_{622} 623) \dots (\log_6 7)(\log_5 6)$?
A. 2 B. 2.5 C. 4 D. 5 E. 6
9. The letters AMATYC are written in order, one letter to a square of graph paper, to fill 100 squares. If three squares are chosen at random without replacement, find the probability to the nearest $1/10$ of a percent of getting three A's.
A. 3.3% B. 3.7% C. 4.0% D. 7.3% E. 11.1%
10. A student committee must consist of two seniors and three juniors. Five seniors are able to serve on the committee. What is the least number of junior volunteers needed if the selectors want at least 600 different possible ways to pick the committee?
A. 6 B. 7 C. 8 D. 9 E. 10
11. Ed drives to work at a constant speed S . One day he is halfway to work when he immediately turns around, speeds up by 8 mph, and drives home. As soon as he is home, he turns around and drives to work at 6 mph faster than he drove home. His total driving time is exactly 67% greater than usual. Find S in mph and write the answer in the corresponding blank on the answer sheet.

12. Each bag to be loaded onto a plane weighs either 12, 18, or 22 lb. If the plane is carrying exactly 1000 lb of luggage, what is the largest number of bags it could be carrying?
 A. 80 B. 81 C. 82 D. 83 E. 84
13. An 8x8 checkerboard is exactly covered by 16  shaped tiles. What is the least possible number of tiles for which the  is horizontal?
 A. 0 B. 2 C. 4 D. 6 E. 8
14. Call a positive integer *biprime* if it is the product of exactly two distinct primes (thus 6 and 15 are biprime, but 9 and 12 are not). If N is the smallest number such that N , $N + 1$, and $N + 2$ are all biprime, find the largest prime factor of $N(N + 1)(N + 2)$.
 A. 13 B. 17 C. 29 D. 43 E. 47
15. You have 8 identical red counters and n identical green counters. You find that you can line them up in a single row in such a way that the number of counters whose right-hand neighbor is the same color equals the number of counters whose right-hand neighbor is the other color. What is the largest possible value of n ?
 A. 17 B. 19 C. 21 D. 25 E. 27
16. If b and c are positive integers such that $b/11$, c/b , and $c/15$ all lie in the interval $(1.5, 1.8)$, find $b + c$.
 A. 43 B. 44 C. 45 D. 46 E. 47
17. Let r , s , and t be nonnegative integers. For how many such triples (r, s, t) satisfying the system $\begin{cases} rs + t = 24 \\ r + st = 24 \end{cases}$ is it true that $r + s + t = 25$?
 A. 23 B. 24 C. 25 D. 26 E. 27
18. In $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. The perpendicular distances from a point P in the interior of $\triangle ABC$ to each of the three sides are equal. Find this distance.
 A. $\frac{9}{2}$ B. $\frac{19}{4}$ C. 5 D. $\frac{21}{4}$ E. $\frac{11}{2}$
19. The digits 1 to 9 can be separated into 3 disjoint sets of 3 digits each so that the digits in each set can be arranged to form a 3-digit perfect square. Find the last two digits of the sum of these three perfect squares.
 A. 26 B. 29 C. 34 D. 46 E. 74
20. The sequence $\{a_n\}$ is defined by $a_0 = a_1 = a_2 = 1$, and $a_{n-3}a_n - a_{n-2}a_{n-1} = (n - 3)!$ for $n \geq 3$. If 5^k is the largest power of 5 that is a factor of $a_{100}a_{101}$, find k .
 A. 20 B. 22 C. 24 D. 25 E. 26

	SPRING 2008
1	E
2	A
3	B
4	E
5	D
6	E
7	E
8	C
9	B
10	D
11	42
12	C
13	C
14	B
15	D
16	A
17	D
18	D
19	E
20	C