

1. The numerical values of the x - and y -intercepts of a straight line are the same nonzero number. What is the slope of the line?
- A. -2 B. -1 C. 0 D. 1 E. 2
2. A fraction is chosen at random from all positive unreduced proper fractions with denominators less than 6. Find the probability that the fraction's decimal representation terminates.
- A. $\frac{3}{5}$ B. $\frac{2}{3}$ C. $\frac{7}{10}$ D. $\frac{7}{9}$ E. $\frac{4}{5}$
3. Two adjacent faces of a rectangular box have areas 36 and 63. If all three dimensions are positive integers, find the ratio of the largest possible volume of the box to the smallest possible volume.
- A. 1 B. 2 C. 3 D. 9 E. 12
4. Five students enroll in a statistics class. The first test is scored on a percent basis (0% to 100%) rounding each score to the nearest whole number. Four of their scores are 93, 96, 99, and 100. How many possible whole number scores on the fifth student's test will make the median of the five scores equal to the mean of the five scores?
- A. 0 B. 1 C. 2 D. 3 E. more than 3
5. In the expression (AM)(AT)(YC), each different letter is replaced by a different digit 0 to 9 to form three two-digit numbers. If the product is to be as large as possible, what are the last two digits of the product?
- A. 20 B. 40 C. 50 D. 60 E. 90
6. A basketball player has a constant probability of 80% of making a free throw. Find the probability that her next successful free throw is the third or fourth one she attempts.
- A. 0.032 B. 0.0384 C. 0.048 D. 0.096 E. 0.192
7. If $\frac{\boxed{a}}{\boxed{c}} \cdot \frac{\boxed{b}\boxed{}\boxed{5}}{\boxed{d}\boxed{}\boxed{3}} = \frac{\boxed{10}\boxed{}\boxed{}}{6} = \frac{\boxed{}}{\boxed{}}$, find the smallest possible value of $a + b + c + d$, if a, b, c , and d are all positive integers.
- A. 4 B. 8 C. 12 D. 16 E. 24
8. Sue works weekdays for \$10 an hour, Saturdays for \$15 an hour, and Sundays for \$20 an hour. If she worked 180 hours last month and earned \$2315, how many more weekday hours than Sunday hours did she work last month?
- A. 75 B. 77 C. 80 D. 82 E. 85
9. Let $s(x) = \sin(\pi x)$ and $S(x) = [s(x)]^2$. Find $s(s(1/6)) + S(S(1/3))$.
- A. $\frac{3}{4}$ B. 1 C. $\frac{4}{3}$ D. $\frac{3}{2}$ E. 2
10. The year 2006 is the product of exactly three distinct primes p, q , and r . How many other years are also the product of three distinct primes with sum equal to $p + q + r$?
- A. 2 B. 3 C. 4 D. 5 E. 6

11. How many positive integers less than 1000 are relatively prime to 105? Two integers are relatively prime if their greatest common divisor is 1.
- A. 325 B. 457 C. 466 D. 533 E. 674
12. In isosceles $\triangle ABE$ with base AB , $AB = 10$ and $BE = 13$. Square $ABCD$ intersects $\triangle ABE$ at points F and G . Find the area common to the interiors of the square and the triangle.
- A. $125/3$ B. 35 C. 40 D. $175/3$ E. 65
13. The equation $x^{\log_{25} 9} + 9^{\log_{25} x} = 54$ has a solution in common with which of the following?
- A. $x^3 - 125x^2 - x + 125 = 0$ B. $x^3 + 5x^2 - 25x - 125 = 0$
C. $x^3 - 5x^2 - 25x + 125 = 0$ D. $5x^3 + 5x^2 - 125x - 125 = 0$
E. $5x^3 - 5x^2 - 125x + 125 = 0$
14. If you have eight pairs of socks, each pair a different color, find the probability that if you randomly lose five socks, the remaining socks form exactly four matching pairs (and three unmatched socks).
- A. $20/39$ B. $7/13$ C. $22/39$ D. $23/39$ E. $8/13$
15. If $h(x) = 2x + 2$ and $k(x) = 2x^3 - 7x^2 - 11x + 6$, find the sum of all of the irrational zeros of $h(k(x))$ and $k(h(x))$.
- A. $1/2$ B. $3/2$ C. $7/2$ D. $9/2$ E. $11/2$
16. If $h(x) = 2x + 2$ and $k(x) = 2x^3 - 7x^2 - 11x + 6$, find the sum of all of the rational zeros of $h(k(x))$ and $k(h(x))$.
- A. $-5/4$ B. $-3/4$ C. $-1/4$ D. $1/4$ E. $3/4$
17. In pentagon $AMTYC$, $AC = MT = 10$, $YT = CY = 20$, $\angle A = \angle M = 135^\circ$, and $\angle Y = 150^\circ$. Find the area of the pentagon to the nearest square unit.
- A. 315 B. 318 C. 320 D. 323 E. 325
18. How many 4-digit numbers whose digits are all odd are multiples of 11?
- A. 80 B. 85 C. 90 D. 95 E. 100
19. Find the tens digit of 3^{2007} .
- A. 0 B. 2 C. 4 D. 6 E. 8
20. In the sequence a_1, a_2, a_3, \dots , $a_1 = 1$, $a_2 = 2$, $a_3 = 5$, and for all $n \geq 3$, $a_{n-1}a_{n-2} = 2a_n a_{n-2} - 2a_{n-1}a_{n-1}$. Find a_{2006}/a_{2005} .
- A. 1002 B. 1002.5 C. 1003 D. 1003.5 E. 1004

1. If $f(x) = 3x - 1$ and $g(x) = x^2$, find $g(f(f(1)))$.
- A. 1 B. 4 C. 5 D. 16 E. 25
2. The teachers at Oak Tech have cars with average mileage 39000 miles. George buys a brand-new car, keeping his old car, and the average mileage drops to 36400. How many cars do the teachers now own?
- A. 12 B. 13 C. 14 D. 15 E. 16
3. The sequence $\log x, \log x^2, \log x^3, \log x^4, \dots$ is best described as which of the following?
- A. geometric with common ratio $\log x$ B. geometric with common ratio x
C. arithmetic with common difference $\log x$ D. arithmetic with common difference x
E. neither geometric nor arithmetic
4. A set of seven different positive integers has mean and median both equal to 20. What is the largest possible value this set can contain?
- A. 65 B. 67 C. 71 D. 73 E. 77
5. If $AM/AT = .YC$, where each letter represents a different digit, AM/AT is in simplest terms, and $A \neq 0$, then $AT =$
- A. 15 B. 16 C. 25 D. 28 E. 75
6. A sheet of stamps is five stamps high and four stamps wide. Each stamp is 2 inches wide and 1 inch high. If a connected group of five stamps is torn from the sheet, let P be the largest possible perimeter and p the smallest possible perimeter for the torn-out group. Find P/p .
- A. $\frac{4}{3}$ B. $\frac{11}{8}$ C. $\frac{11}{7}$ D. $\frac{10}{7}$ E. $\frac{3}{2}$
7. If $\ln s = 0.6$ and $\ln t = 0.9$, find $\log_{st} e^{5.4}$.
- A. 3.6 B. 5 C. 5.4 D. 10 E. 10.8
8. A function f is symmetric to the origin and periodic with period 8. If $f(2) = 3$, what is the value of $f(4) + f(6)$?
- A. -6 B. -3 C. 0 D. 3 E. 6
9. For how many integer values of k do the graphs of $x + y = k$ and $xy = k$ NOT intersect?
- A. 0 B. 1 C. 2 D. 3 E. more than 3
10. A point is chosen at random from the interior of a square of side 16. Find the probability that the point is at least $\sqrt{2}$ units from both diagonals.
- A. $\frac{9}{16}$ B. $\frac{5}{8}$ C. $\frac{3}{5}$ D. $\frac{3}{4}$ E. $\frac{4}{9}$
11. The graph of the function $f(x) = x + \sin kx$ ($|k| \leq 1$) intersects the graph of the function $f^{-1}(x)$ at $(4,a)$, $(12,b)$, and $(-8,c)$. Find the value of $a + b + c$.
- A. 4 B. 8 C. 10 D. 12 E. 16

12. If $\cos(\arctan(x)) = x$ (x in radians), then x^2 can be expressed in the form $\frac{a + \sqrt{b}}{2}$. Find $a + b$.
 A. 4 B. 5 C. 6 D. 7 E. 8
13. A jug holds 10 gal of antifreeze. I fill an empty bottle from the jug and replace the amount I poured out with water, mixing well. I refill the emptied bottle again from the jug, refilling the jug with water and mixing well, and then repeat this process once more. The bottle is now half water. To the nearest tenth of a gallon, what is the volume of the bottle?
 A. 2.3 B. 2.5 C. 2.7 D. 2.9 E. 3.1
14. How many different 3-letter strings can be formed from the letters of MATHEMATICS (no letter can be used in a given string more times than it appears in the word)?
 A. 336 B. 399 C. 660 D. 675 E. 990
15. A farmer plants A acres of wheat one year. Each year thereafter, he harvests (removes) $1/4$ of the planted acreage and then plants 1500 more acres. The number of acres of wheat planted approaches what number?
 A. 3000 B. 4000 C. 5000 D. 6000 E. it depends on A
16. Right $\triangle ABC$ (right angle at B) has legs of length 68 and 285. If the medians from vertex A and vertex C intersect at D , find the area of $\triangle ADC$ to the nearest ten square units.
 A. 3220 B. 3230 C. 3240 D. 3250 E. 3260
17. If $f(x) = \frac{x^2 - 3x - 4}{x + 1}$, the inverse of $f(x)$ can be written as $f^{-1}(x) = \frac{x^2 + ax + b}{x + c}$. Find $a + b + c$.
 A. -14 B. -2 C. 4 D. 10 E. 34
18. Choose k so that the system
$$\begin{cases} x + y + kz = 1 \\ x + ky + z = 2 \\ kx + y + z = 3 \end{cases}$$
 is dependent. For which pair (x,y) below does there exist a z such that (x,y,z) will satisfy the resulting dependent system?
 A. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}, 0$ B. $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \frac{2}{3}$ C. $\begin{bmatrix} 8 \\ 3 \end{bmatrix}, 1$ D. $\begin{bmatrix} 4 \\ 3 \end{bmatrix}, 1$ E. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, 2$
19. A pentagon is circumscribed about a circle of diameter 6 (that is, each side of the pentagon is tangent to the circle). If the pentagon has area 42 cm^2 , find its perimeter in centimeters.
 A. 14 B. 21 C. 24 D. 28 E. 35
20. The sum of the solutions of $\arctan \frac{1}{x} + \arctan \frac{1}{x+2} = \arctan \frac{4}{x+4}$ is
 A. negative B. even C. 1 D. greater than 5 E. prime

PLEASE PRINT!

AMATYC STUDENT MATHEMATICS LEAGUE

COLLEGE: KEY - FALL 2006 STATE: _____

LAST NAME:

GENDER: Male Female

FIRST NAME:

EMAIL ADDRESS: _____ INSTRUCTOR _____

DO YOU HAVE A TWO-YEAR COLLEGE OR HIGHER DEGREE FROM ANY SCHOOL IN THE WORLD? YES NO

	Student's Responses	Local Corrector
1	B	
2	E	
3	D	
4	C	
5	B	
6	B	
7	D	
8	B	
9	D	
10	C	
11	B	
12	D	
13	A	
14	A	
15	C	
16	A	
17	D	
18	B	
19	E	
20	E	

ROUND: 1 2

correct = _____

incorrect = _____

blank = _____

= # correct $\times 2$

— = # wrong $\times \frac{1}{2}$

 = score

PLEASE PRINT!

AMATYC STUDENT MATHEMATICS LEAGUE

COLLEGE: KEY – SPRING 2007 (CORRECTED) STATE: _____

LAST NAME:

GENDER: Male Female

FIRST NAME:

EMAIL ADDRESS: _____ INSTRUCTOR _____

DO YOU HAVE A TWO-YEAR COLLEGE OR HIGHER DEGREE FROM ANY SCHOOL IN THE WORLD? YES NO

	Student's Responses	Local Corrector
1	E	
2	D	
3	C	
4	C	
5	C	
6	D	
7	A	
8	B	
9	D	
10	A	
11	B	
12	A	
13	D	
14	B	
15	D	
16	B	
17	E	
18	C	
19	D	
20	E	

ROUND: 1 2

correct = _____

incorrect = _____

blank = _____

= # correct × 2

— = # wrong × $\frac{1}{2}$

_____ = score