

1. If  $L$  has equation  $ax + by = c$ ,  $M$  is its reflection across the  $y$ -axis, and  $N$  is its reflection across the  $x$ -axis, which of the following must be true about  $M$  and  $N$  for all nonzero choices of  $a$ ,  $b$ , and  $c$ ?

- A. the  $x$ -intercepts are equal      B. the  $y$ -intercepts are equal      C. the slopes are equal  
D. the slopes are opposite      E. the slopes are reciprocals

2. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2?

- A. \$1.64    B. \$1.78    C. \$1.86    D. \$1.89    E. \$1.99

3. When the polynomial  $P(x)$  is divided by  $(x-2)^2$ , the remainder is  $3x - 3$ . What is the remainder when  $(x-1)P(x)$  is divided by  $(x-1)(x-2)^2$ ?

- A.  $3x - 3$     B.  $3x^2 - 6x + 3$     C. 3    D.  $x - 1$     E.  $x - 2$

4. If  $f(x) = 3x - 2$ , find  $f(f(f(3)))$ .    A. 19    B. 55    C. 75    D. 107    E. 163

5. What is the remainder when  $x^3 - 2x^2 + 4$  is divided by  $x + 2$ ?

- A. -12    B. 0    C. 4    D. 6    E. 12

6. Let  $p$  be a prime number and  $k$  an integer such that  $x^2 + kx + p = 0$  has two positive integer solutions. What is the value of  $k + p$ ?

- A. 1    B. -1    C. 0    D. 2    E. -2

7. What is the least number of prime numbers (not necessarily different) that 3185 must be multiplied by so that the product is a perfect cube?

- A. 1    B. 2    C. 3    D. 4    E. 5

8. Two adjacent faces of a three-dimensional rectangular box have areas 24 and 36. If the length, width, and height of the box are all integers, how many different volumes are possible for the box?

- A. 2    B. 3    C. 4    D. 5    E. 6

9.  $(\tan t - \sin t \cos t) / (\tan t) =$

- A.  $\sin t$     B.  $\cos t$     C.  $\sin^2 t$     D.  $\cos^2 t$     E. 1

10. The counting numbers are written in the pattern at the right. Find the middle number of the 40<sup>th</sup> row.

			1			
			2	3	4	
			5	6	7	8
			10	11	12	13
				14	15	16

- A. 1561    B. 1641    C. 1559    D. 1639    E. 1483

11. The solution set of  $x^2 - 3x - 18 \geq 0$  is a subset of the solution set of which of the following inequalities?

- A.  $x^2 - x - 20 \geq 0$       B.  $(x - 4)/(x + 3) \geq 0$       C.  $x^2 - 8x + 14 \geq 0$   
D. both B and C      E. all of A, B, and C

12. If  $2a - 4b = 128b^3 - 16a^3$  and  $a \neq 2b$ , find  $a^2 + 2ab + 4b^2$ .

- A.  $-1/8$       B.  $-1/2$       C.  $1/2$       D.  $1/8$       E.  $2$

13. Square ABCD is inscribed in circle O (that is, A, B, C, and D all lie on the circle) and its area is  $a$ . Square EFGH is inscribed in a semicircle of circle O (that is, E and F lie on a diameter and G and H lie on the circle). What is the area of square EFGH?

- A.  $a/5$       B.  $2a/5$       C.  $a/3$       D.  $a/2$       E.  $3a/5$

14. Consider all arrangements of the letters AMATYC with either the A's together or the A's on the ends. What fraction of all possible such arrangements satisfies these conditions?

- A.  $1/5$       B.  $2/15$       C.  $1/3$       D.  $2/5$       E.  $3/5$

15. The year 2003 is prime, but its reversal, 3002, is not. In fact, 3002 is the product of exactly three different primes. Let  $N$  be the sum of these three primes. How many other positive integers are the products of exactly three different primes with this sum  $N$ ?

- A.  $0$       B.  $1$       C.  $2$       D.  $3$       E.  $4$

16. In a group of 30 students, 25 are taking math, 22 English, and 19 history. If the largest and smallest number who could be taking all three courses are  $M$  and  $m$  respectively, find  $M + m$ .

- A.  $17$       B.  $19$       C.  $22$       D.  $23$       E.  $25$

17. A boat with an ill passenger is  $7\frac{1}{2}$  mi north of a straight coastline which runs east and west. A hospital on the coast is 60 miles from the point on shore south of the boat. If the boat starts toward shore at 15 mph at the same time an ambulance leaves the hospital at 60 mph and meets the ambulance, what is the total distance (to the nearest 0.5 mile) traveled by the boat and the ambulance?

- A.  $60.5$       B.  $61$       C.  $61.5$       D.  $62$       E.  $62.5$

18. If each letter in the equation  $\sqrt{AMATYC} = MYM$  represents a different decimal digit, find  $T$ 's value.

- A.  $3$       B.  $4$       C.  $5$       D.  $6$       E.  $7$

19. If  $a$ ,  $b$ ,  $c$ , and  $d$  are nonzero numbers such that  $c$  and  $d$  are solutions of  $x^2 + ax + b = 0$  and  $a$  and  $b$  are solutions of  $x^2 + cx + d = 0$ , find  $a + b + c + d$ .

- A.  $-2$       B.  $-1$       C.  $0$       D.  $1$       E.  $2$

20. Al and Bob are at opposite ends of a diameter of a silo in the shape of a tall right circular cylinder with radius 150 ft. Al is due west of Bob. Al begins walking along the edge of the silo at 6 ft per second at the same moment that Bob begins to walk due east at the same speed. The value closest to the time in seconds when Al first can see Bob is

- A.  $46$       B.  $47$       C.  $48$       D.  $49$       E.  $50$

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NAME: \_\_\_\_\_ KEY -- Nov 2003 \_\_\_\_\_

COLLEGE: \_\_\_\_\_


ROUND: 1 2


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1	<b>C</b>	
2	<b>A</b>	
3	<b>B</b>	
4	<b>B</b>	
5	<b>A</b>	
6	<b>B</b>	
7	<b>E</b>	
8	<b>E</b>	
9	<b>C</b>	
10	<b>A</b>	
11	<b>C</b>	
12	<b>A</b>	
13	<b>B</b>	
14	<b>D</b>	
15	<b>C</b>	
16	<b>E</b>	
17	<b>E</b>	
18	<b>E</b>	
19	<b>A</b>	
20	<b>C</b>	


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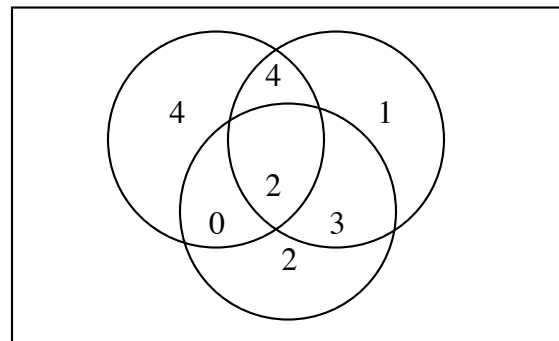
 = # correct  $\square$  2

-  = # incorrect  $\square$   $\frac{1}{2}$

 = score

1. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value?  
 A. 60% B. 120% C. 150% D. 200% E. 300%
2. Which of the following is NOT a factor of  $x^4 - 4x^3 - x^2 + 16x - 12$ ?  
 A.  $x - 2$  B.  $x + 2$  C.  $x - 1$  D.  $x + 1$  E.  $x - 3$
3. The library in Johnson City has between 1000 and 2000 books. Of these, 25% are fiction,  $1/13$  are biographies, and  $1/17$  are atlases. How many books are either biographies or atlases?  
 A. 240 B. 250 C. 270 D. 280 E. 300
4. A trecimal is like a decimal, except the digits represent fractions with powers of 3 instead of 10. For example,  $16/27 = 1/3 + 2/9 + 1/27 = 0.121$  as a trecimal. How is  $77/81$  expressed as a trecimal?  
 A. 0.950617 B. 0.2012 C. 0.1211 D. 0.1111 E. 0.2212
5. The function  $P(t) = \cos 8t$  can be written as sums and differences of powers of  $\cos t$ . When  $P(t)$  is written that way, what is the coefficient of  $(\cos t)^3$ ?  
 A. 0 B. 1 C. -1 D. 2 E. -2
6. If  $\log_a b = 64$ , find  $\log_{a^2} b^3$ .  
 A. 16 B. 48 C.  $128/3$  D. 96 E. 512
7. The number  $877530p765q6$  is divisible by both 8 and 11, with  $p$  and  $q$  both digits from 0 to 9. The number is also divisible by  
 A. 7 B. 12 C. 16 D. 18 E. not enough information to know
8. Teams A and B play a series of games; whoever wins two games first wins the series. If Team A has a 70% chance of winning any single game, what is the probability that Team A wins the series?  
 A. 0.616 B. 0.637 C. 0.657 D. 0.700 E. 0.784

9. The Venn diagram at the right represents sets A, B and C (not necessarily in that order). Depending on how the diagram is labeled, how many different answers are possible for the number of elements in the set  $A - B$ ? (Note:  $A - B$  is all elements which are in A but not in B)



- A. 2 B. 3 C. 4 D. 5 E. 6

10. A fixed point for a function  $y = f(x)$  is a real number  $r$  such that  $f(r) = r$ . How many of the following functions must have a fixed point?

polynomial function of odd degree  $> 1$   
 trigonometric function  $y = A \sin Bx + D$

polynomial function of even degree  $> 0$   
 rational function  $y = (x - a)/(x - b)$

- A. 0 B. 1 C. 2 D. 3 E. 4

11. Which of the following is the identity function  $f(x) = x$  for all real numbers?

- A.  $e^{\ln x}$  B.  $\ln e^x$  C.  $\sin(\arcsin x)$  D.  $\arctan(\tan x)$  E.  $\sqrt{x^2}$

12. A circular table is pushed into a corner of a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radius of the table?
- A. 5 inches      B. 12 inches      C. 15 inches      D. 17 inches      E. 20 inches
13. In  $\triangle ABC$ ,  $\angle C = 90^\circ$  and  $\cos \angle A = 4/5$ . If D is the midpoint of side AC, find  $\cos \angle CDB$ .
- A.  $\frac{2\sqrt{13}}{13}$       B.  $\frac{5}{9}$       C.  $\frac{\sqrt{5}}{4}$       D.  $\frac{2}{5}$       E.  $\frac{3}{5}$
14. Enrique walks along a level road and then up a hill. At the top he immediately turns and walks back to his starting point. He walks 4 mph on level ground, 3 mph uphill, and 6 mph downhill. If the entire walk takes 6 hours, how far does he walk?
- A. 16 mi      B. 20 mi      C. 24 mi      D. 28 mi      E. 32 mi
15. If  $x^2 = x + 3$ , then  $x^3 =$
- A.  $x + 6$       B.  $4x + 3$       C.  $4x^2 + 3$       D.  $x^2 + 3x + 3$       E.  $x^2 + 27$
16. A bag holds 5 cards identical except for color. Two are red on both sides, two are black on both sides, and one is red on one side and black on the other. If you pick a card at random and see that the only side you can see is red, what is the probability that the other side is also red?
- A.  $1/2$       B.  $2/3$       C.  $3/4$       D.  $4/5$       E.  $5/6$
17. The set S contains the number 2, and if it contains the number n, it also contains  $3n$  and  $n + 5$  (assume S contains only numbers produced by these rules). Which of the following is NOT in S?
- A. 2000      B. 2001      C. 2002      D. 2003      E. 2004
18. Let  $f(x) = ax + b$ , with  $b < a$  both positive integers. If for positive integers p and q,  $f(p) = 18$  and  $f(q) = 39$ , what is the value of b?
- A. 1      B. 3      C. 4      D. 7      E. 8
19. In  $\triangle SBC$ ,  $SB = 12$ ,  $BC = 15$ , and  $SC = 18$ . Let O be the point for which BO bisects angle SBC and CO bisects angle SCB. If M and L are on sides SB and SC respectively so that ML is parallel to side BC and contains point O, what is the perimeter of  $\triangle SML$ ?
- A. 24      B. 27      C. 30      D. 32      E. 36
20. Ed has four children, Al, Bo, Cy, and Di (in order oldest to youngest). Bo is 4 years older than Cy and 12 years older than Di. This year Ed notices that he is twice as old as Bo, and the sum of the squares of the children's ages equals the square of Ed's age. If Di just became a math teacher, what is the sum of the children's ages?
- A. 48      B. 76      C. 100      D. 128      E. 148

ANSWERS:

- |     |   |     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|-----|---|
| 1.  | C | 2.  | D | 3.  | A | 4.  | E | 5.  | A |
| 6.  | D | 7.  | B | 8.  | E | 9.  | C | 10. | C |
| 11. | B | 12. | D | 13. | A | 14. | C | 15. | B |
| 16. | D | 17. | A | 18. | C | 19. | C | 20. | E |